## QM 250

Mid-term Exam

## Revision

## Example sheet

1. The ages, in years, of sample of 7 employees, chosen at random from UOB staff are as follow :


## A) Fined the

- Mean, median, mode
- Range, standard deviation, variance
- $70^{\text {th }}$ percentile and interpret its meaning
- Coefficient of skewness . is the distribution of the data symmetric
b) Consider the above values a population. fined the mean , standard deviation ,variance of this population .

2. The following are the number of minutes to commute from home to college for a sample of 10 students :

| 30 | 25 | 19 | 40 | 20 | 32 | 35 | 28 | 36 | 26 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

A) Group the above data into classes using a frequency distribution table .
B) Fined the mean, and the standard deviation
C) Draw a histogram

1. You are given the following sample information . fined the mean and the standard deviation .

Class limits:
18 up to $<24 \quad 24$ up to $<30 \quad 30$ up to $<36$

No. of item:
8
5
2

## CH 1 : Introduction to Statistics

## Definition of statistics:

A science that is concerned with the collection, or organization, Analysis, and interpretation ofdata to make better decision

Function of statistics:
1- Collection
2- Organization
3- Analysis
4- Interpretation
Data can be collected from
different sources
depending on the:
Type of data and the purpose of the data . or which
the data are collected.
Data can be collected either from a population or
from a sample

Population : The number of all items or objects which are of interest to our study

Sample : small part of population

## The reasons why we study a sample and not a population:

1. The study of population will need a large amount of resources, time, effort, money
2. The population is too big to study, and even too different to fined
3. The study of population could destroy population

Not any sample is a good sample, because you need a sample that represent a population

In order for the sample to be random, every item or number of the population should have anequal chance of being chosen in the sample

Whether data are collected from the population or from the sample, they are aboutsomething called a variable

## Types of variable:

1. Qualitative ( non-numeric )

Examples: Gender, colors, professions ( job ) , name of places
2. Quantitative (numeric)
A) Discrete variable:

Example: number of family
B) Continues variable:

Example: weigh, clock, size, temperature
most of the variable in our life are continuous

## CH2,3,4

## Organization of data:

After data are collected, they need to be organized in a useful, meaningful form , in chapter $(2+3+4)$ we will study different ways of organizing data , data can be : ungrouped data, grouped data.

## 1. Organizing un-grouped sample data:

In this case we summaries the data in different ways without grouping them into certaingroups.

The mean : is the sum ( total ) of all data values divided by their number.

## Equation $\overline{: x}=\frac{\sum x}{n}$

$\bar{x}$ : mean of x value
$X$ : each value in the data
n : the number of data values ( sample size )
$\sum$ : the sum of values called ( sigma )

## Example (1) : from the examples sheet :

$\bar{X}=\frac{\sum(44+38+25+32+27+40+25)}{7}$
$\bar{x}=\frac{231}{7}=33$ years

## The features of the mean :

1. For every set of data there is only ( one ) mean
2. All data values are used in the calculation of the mean (every item is used)
3. The mean is also known as the average, arithmetic mean, and the expected value.
4. The mean is influenced ( affected) by extreme values, we mean the value that is either too big or too small compared to our data.
therefore, if we have an extreme value in the data, it is better to not use themean .
5. The sum of the deviation between the data values and their mean is always (zero).

Example for point 5 ■

| x | $x-\bar{x}$ |
| :---: | :---: |
| 10 | $10-9=1$ |
| 12 | 12-9 = 3 |
| 8 | $8-9=-1$ |
| 6 | $6-9=-3$ |
| $\sum_{36}^{x}=$ | $\begin{gathered} \sum(x-\bar{x})=0 \\ \Sigma(1+3-1-3)= \\ 0 \end{gathered}$ |
| $\bar{x}=9$ |  |

## The median :

is the middle ( or the central ) value of an ordered set of data . by ordered set of data, we mean the data values have to be rearranged starting from the smallest and ending with the largest

## Example (1) : from the examples sheet

First we rearranged the values from the smallest to

## biggest

$25-25-27-32-38-40-44$
The median is 32 years
if we have even numbers of value we take the 2
central values and divided them by 2

## Example :

$90-100-110-120-125-13$

$$
\frac{110+120}{2}=115
$$

The median is 115
we can find the median using the
location equation :-Location of the
median
$=0.5(n+1)$
$=0.5(7+1)=4$
4 is not the median, it is the location
of the median The forth value from the
sample data is the media $=32$

## The features of the median

1. $50 \%$ of the data values come before the median, and the other 50 \% are after themedian.
2. The median is not affected by extreme values in the data.
3. For every set of data there is only (one ) median.

The mode : is the value or values that occurs more frequently in the data than other values

## Example (a)

The mode is 25 year

## The features of the mode:

1. For any set of data, there could be no mode, one mode, or more than one mode

## Example:

| values |  |  |  |  |  |  |  | mode | Reason |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 120 | 170 | 90 |  |  |  |  | Non | No value repeated |
| 9 | 7 | 9 | 7 | 9 | 7 |  |  | Non | The values are <br> repeated equally |
| 35 | 37 | 90 | 40 | 37 | 80 |  |  | 37 |  |
| 80 | 89 | 90 | 89 | 90 | 70 | 30 | 90 | $89 \&$ <br> 90 |  |

2. The mode is not affected by extreme values
3. The mode can be used to describe quantitative as well as qualitative

## Example:

| black | red | blue | black | green |
| :---: | :---: | :---: | :---: | :---: |

## The mode is black

The Mean , the Median , and Mode are known as : numerical measures of location

## Example:

The range : is the difference between the highest data values and the lowest
. Range $=\mathrm{H}-\mathrm{L}$
= 44-25
= 19 years

## The features of the range:

1. The range is affected by extreme value
2. It is a weak measure of variation, because it depends only on 2 values of the data andignore the rest
3. There is only ( one ) range for the set of data

The standard deviation measures how far the data values are from their mean .
Equation: $\mathbf{s}=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}$

S: the sample standard deviation
$X$ : every item in the data
$\bar{X}$ : the sample mean
N : the sample size

## Example :

the standard deviation

To solve any question with the standard deviation equation we use this table :

| x | $X-\bar{x}$ | $(x-\bar{x})^{2}$ |
| :---: | :---: | :---: |
| 44 | $44-33=11$ | $(11)^{2}=121$ |
| 38 | $38-33=5$ | $(5)^{2}=25$ |
| 25 | $25-33=-8$ | $(-8)^{2}=64$ |
| 32 | $32-33=-1$ | $(-1)^{2}=1$ |
| 27 | $27-33=-6$ | $(-6)^{2}=36$ |
| 40 | $40-33=7$ | $(7)^{2}=49$ |
| 25 | $25-33=-8$ | $(-8)^{2}=64$ |
| $\sum x=231$ | $\sum(x-\bar{x})=0$ |  |
| $\bar{x}=\begin{gathered} \sum x \\ n \end{gathered}=\frac{231}{7}=33$ | To be sure that your answer is correct the sum of deviation between the data values and their means is always (zero ) | $\sum(x-\bar{x})^{2}=360$ |

$S=\sqrt{\frac{360}{7-1}}=7.75$

## The variance : is the standard deviation squared

$$
S^{2}=(7.75)^{2}=60
$$

$S^{2}$ : the sample variance
\# the standard deviation is the
square root of the variance\# the
bigger is the variance

## Features of variance \& standard deviation:

1. Cannot be (negative), but it can be ( positive) or (Zero )
2. If the standard deviation is (zero) this means there is no difference or deviation between the data values and there means this happens when the data values are allequal and the same
3. The standard deviation is influenced by extreme value4- The smaller standard deviation is the better
*the range, standard deviation, and variance known as : The numerical measures of variation \#*variation is also known as : deviation , difference , change , spread , dispersion

The percentiles : are the values that divide an order set of data into 100 equal parts .
*To find a percentiles we have to :
1 Rearrange values from smallest to largest
2. Fined the location of the percentiles
3. The value of the location

## Example:

Rearrange values from smallest to largest

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 25 | 27 | 32 | 38 | 40 | 44 |

## Location of $P_{70}=0.7(n+1)$

$=0.7(7+1)=5.6 \ggg$ the location (it means the value of $P_{70}$ is between the $5^{\text {th }}$ and $6^{\text {th }}$ values )

## Value of $P_{70}$

$=1^{\text {st }}$ value + the fraction ( $2^{\text {nd }}$ value $-1^{\text {st }}$ value )
$=5^{\text {th }}$ value $+0.6\left(6^{\text {th }}-5^{\text {th }}\right)$
$=38+0.6(40-38$
$=P_{70}=39.2$
This means that $70 \%$ of the data values are less than 39.2
$P_{50}=0.5(n+1)$ the location of $P_{50}$ is the same location of the median .

1) $P_{25}$ : is known as the first quartile ( $Q_{1}$ )
2) $P_{50}$ : is known as the second quartile ( $Q_{2}$ )
3) $P_{75}$ : is known as the third quartile ( $Q_{3}$ )
4) Quartiles are values that divide an ordered set of data into 4 equal parts.
5) $25 \%$ of the data values falls below $Q_{1}, 50 \%$ of the data values come before $Q_{2}$, and $75 \%$ of the data values come before $Q_{3}$.

## Location of

$Q_{1}=0.25$
( $\mathrm{n}+1$ )
Location of
$Q_{2}=0.5$
( $\mathrm{n}+1$ )
Location of
$Q_{3}=0.75$
( $\mathrm{n}+1$ )
*Percentiles and Quartiles are not influenced
by extreme values\# In total we have 3
Quartiles and 99 Percentiles

## Coefficient skewness ( sk ):

There are 3 types of skewness : positive, negative , normal

*Mode : the value that have the highest point

1. In the Positive skewness :

Mean < Median < Mode
2. In the negative skewness:

Mode < Median < Mean
3. In the normal ( symmetric) skewness :

Mean $=$ Median $=$ Mode

## Example:

SK $=\frac{3 \text { (mean-median) }}{\text { stander deviation }}$

$$
=\frac{3(33-32)}{7.75}
$$

## $=+0.39 \lll$ there is a positive skewness in our data

*if the curve has no skewness, sk=0 it means that the curve is symmetric or normal curve

## Features of (SK ) :

1. Sk ranges between -3 and $+3:-3 \leq S K \leq+3$
2. If SK negative, this means there is a negative skewness in data and the curve is turningto the left .
Example (1)
The distribution of our data is not symmetric because $\mathrm{SK} \neq 0$
3. If SK positive , this means there is a positive skewness in data and the curve is turningto the right .
4. Is $\mathrm{SK}=0$, this means there is no skewness in the data and the data curve is normal orsymmetric.

## The numerical measures :

1. Mean 2. Median 3. Mode
2. range 5. Standard deviation 6 . Variance
3. percentile 8. Quartiles 9.Skewness

## Example:

The population mean is
$\mu=\frac{\sum x}{N}$

$$
=\frac{231}{7}=33
$$

The population standard deviation is $\boldsymbol{\sigma}=\sqrt{\frac{\sum(x-\mu)^{2}}{N}}$
to fine the population standard deviation we should make this table :

| x | $\mathrm{X}-\mathrm{M}$ | $(X-\mu)^{2}$ |
| :---: | :---: | :---: |
| 44 | $44-33=11$ | 121 |
| 38 | $38-33=5$ | 25 |
| 25 | $25-33=-8$ | 64 |
| 32 | $32-33=-1$ | 1 |
| 27 | $27-33=-6$ | 36 |
| 40 | $25-33=-8$ | 49 |
| 25 | $\sum(x-\mu)=0$ | $\sum(x-\mu)^{2}=360$ |
| $\sum x=231$ |  | 64 |
| $\mu=33$ |  |  |
| 20 |  |  |

$\sigma=\sqrt{\frac{360}{7}}=7.17$
The population variance $=\sigma^{2}$
$=(7.17)^{2}=51.43$

Equation: $\boldsymbol{\sigma}^{\mathbf{2}}=\frac{\sum((x-\mu))^{2}}{N}$
*The other numerical measures (median , mode, range, quartile , percentile, SK ) are foundthe same way used in sample date .

## Exercise :

You are given the following diagram, fined the following
a) $P_{50}$
b) sk if $S^{2}=196$
c) $Q_{2}$


## Solution :

a) $P_{50}=$

70 , because it is the same as the median .
b) $S K=\frac{3(\text { mean }- \text { median })}{\text { stander devition }}=\frac{3(65-70)}{\sqrt{196}}=-1.07$

There is a negative skewness on the data
c) $Q_{2}=P_{50}=$ Median $=70$
*The data in this question are sample, because of the $S^{2}$

## Exercise 2 :

The following sample : 410-300-420-310-M-430 has a Mean of 375 . fined the $M$ , the Median, $P_{40}, Q_{1}$
A) Mean $=\frac{\sum x}{n}$

$$
375=\frac{\sum x}{6} \text { (طرفين في وسطين) }
$$

$\sum \bar{x}=375 \times 6=2250$
$2250=1870($ sample total $)+M$
2250-1870 = M
$380=\mathrm{M}$
b) Median :

Rearrange data :

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 300 | 310 | 380 | 410 | 420 | 430 |

We will solve it by the
location :-
Location $=0.5(n+1)$
$=0.5(6+1)=3.5$
Median $=3^{\text {rd }}$ value $+0.5\left(4^{\text {th }}-3^{\text {rd }}\right)$
$=380+0.5(410-380)$
395
c) $P_{40}=0.4(\mathrm{n}+1)$
$=0.4(6+1)=2.8 \ggg$ location (between the second and third value )
$P_{40}=310+0.8(380-310)=366$
*We must rearrange the data win fined : Median , Percentile, Quartile
d) $Q_{1}$ :-
$P_{25}=0.25(7)=1.57 \ggg$ the location
$Q_{1}=300+0.75(310-300)=307.5$

## 2. Organizing grouped sample data :

In this case the data values need to be classified or grouped into groups or classes, we dothis using frequency distribution table .

Example ( 2 ) :- from the example sheets
To make a frequency table we follow these steps :

## 1. Determine the number of classes ( $K$ ) by using the following rule :

$2^{K}<n$
try different K :-
$2^{1}=2$
$2^{2}=4$
$2^{3}=8$
$2^{4}=16 \ggg$ the best no. of classes for our data is 4
2. Fined the classes interval ( i ) or width : using the following rule :
$\mathrm{i} \geq \frac{H-l}{k}$
, H : the highest / maximum value in the data
L : the lowest / minimum value in the data
*H-L = Range
$\mathrm{i} \geq \frac{40-19}{4}=5.25 \approx 6$

* if we have a fraction in the value of $i$, we should round the value to the following whole number, even if the fraction 0.01


## Make a frequency table :

| $\begin{aligned} & \mathrm{N} \\ & \mathrm{o} . \end{aligned}$ | Class <br> limits | ta <br> Ily | f | m | $\begin{aligned} & \mathbf{f} \\ & \mathbf{m} \end{aligned}$ | $\begin{aligned} & \text { ( m } \\ & -\lambda \end{aligned}$ | $(m)^{2-x}$ | $\begin{aligned} & (m-x \\ & )^{2} f \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & 19 \text { up to } \\ & 25 \end{aligned}$ | // | 2 | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ | -7.8 | 60.84 | 121.68 |
| 2 | $\begin{gathered} 25 \text { up to } \\ 31 \end{gathered}$ | // | 4 | 2 | $\begin{aligned} & 1 \\ & 1 \\ & 2 \end{aligned}$ | -1.8 | 3.24 | 12.96 |
| 3 | $\begin{aligned} & 31 \text { up to } \\ & 37 \end{aligned}$ | // | 3 | 3 | $\begin{aligned} & 1 \\ & 0 \\ & 2 \end{aligned}$ | 4.2 | 17.64 | 52.92 |
| 4 | 37 up to 43 | / | 1 | 4 0 | $\begin{aligned} & 4 \\ & 0 \end{aligned}$ | 10.2 | 104.04 | 104.04 |
| $\begin{aligned} & \text { to } \\ & \text { tal } \end{aligned}$ |  |  | $\begin{gathered} \sum f= \\ 10 \end{gathered}$ | - | $\begin{aligned} & \sum_{f m} \\ & = \\ & 298 \end{aligned}$ |  |  | $\begin{aligned} & \sum\left(m-^{-}\right)^{2} f \\ & =291.6 \end{aligned}$ |

b) $X=\frac{f m}{n}=\frac{298}{10}=29.8$, you could write the equation as : $\frac{\sum f m}{\sum f}$

## Notes :

1- The lower limit of the $1^{\text {st }}$ class is the same as the lowest data value .

2- Upper limit of class = lower limit + (i)

$$
=19+6=25
$$

3. The lower limit of the following class is the same as the upper limit of the previous one.
4. ( f ) means frequency : frequency means the number of data values that related to a class.
5. 5- The sum of ( f ) is the same of $\mathrm{n}: \sum f=\mathrm{n}$
6. ( m ) means class midpoint $\mathrm{m}, \mathrm{m}=\frac{\text { lower limit }+ \text { upper limit }}{2}$
7. The mean of grouped sample data , $\bar{x}=\frac{\sum f m}{n}$
8. The standard deviation of grouped sample data is:

$$
s=\sqrt{\frac{\sum(m-x)^{2}}{n-1}}=\sqrt{\frac{291.6}{10-1}}=5.69
$$

9. Variance $\left(S^{2}\right)=(5.69)^{2}=32.4$

## C ) draw a histogram :

A histogram is a bar graph drawn between 2 axes, on the vertical axis we show the frequencies, and on the horizontal axis we show the class limits . the bars of the histogram donot have gaps between them .


From a histogram, we find that the majority of the student commute between 25 to 31 minutes . and the majority of them commute between 37 and 43 .

## Example (3):

No. of classes ( $K$ ) = 3
$\mathrm{i}=\mathrm{UL}-\mathrm{LL}=24-18=6$
classes interval $(i)=6$
$\sum f=\mathrm{n}, \mathrm{n}=8+5+2=15$


| No. | Class limits | f | m | fm | ( m- ${ }^{-1}$ | $(m-x)^{2}$ | $\left.(m-)^{-}\right)^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 18 up to 24 | 8 | 21 | 168 | -3.6 | 12.96 | 103.68 |
| 2 | 24 up to 30 | 5 | 27 | 135 | 2.4 | 5.76 | 28.8 |
| 3 | 30 up to 36 | 2 | 33 | 66 | 8.4 | 70.56 | 141.12 |
| total |  | $\sum f=15$ | - | $\begin{aligned} & \sum f m \\ & =369 \end{aligned}$ |  |  | $\begin{aligned} & \sum\left(m-^{-} x^{2} f\right. \\ & =273.6 \end{aligned}$ |

$\bar{x}=\frac{\sum f m}{n}=\frac{369}{15}=24.6$

## The difference between any $\mathbf{2}$ mid-points = the class interval (i)

$\mathrm{S}=\sqrt{\frac{\left.\sum(m-x)\right)^{2} f}{n-1}}=\sqrt{\frac{273.6}{15-1}}=\sqrt{19.54}=4.42$

## Notes:

1. The mean of grouped population data is: $\mu=\frac{\sum f m}{n}$
2. The population standard deviation of grouped sample data is:

$$
\sqrt{\frac{\left.\sum(N-M)\right)^{2} f}{N}}
$$

1. Any numerical measures obtained from a sample data is called : Statistic
2. Any numerical measures obtained from a Population data is called :

Parameter (note : toremember them easily connect the 2 words with the letter >>> sample $=$ Statistics ( S ) Population = Parameter ( P )
3. The science of statistics can be divided into 2 branches:

Descriptive statistics : this consists of all methods used to summarize , organize , and present data .

Inferential data : it consists of all the methods used to make decisions about apopulation based on sample data .

## Exercise :

You are given the following diagram, answer the questions below :


What is the name of the diagram ?
Histogram

How many students take the test ( sample size ) ?
$\sum f=\mathrm{n}=3+9+11+7+5=35$
How much is the class interval ?
>> $\mathrm{i}=\mathrm{UL}(60)-\mathrm{LL}(50)=10$

How many classes of data are used ?
classes, because each par represent a class

What is the class mid-point for the second class ?
$\gg \mathrm{m}=\frac{u l+l l}{2}=\frac{70+60}{2}=65$
How many students scored less than $\mathbf{8 0}$ ?
$11+9+3=23$

How many students failed (<60) ?
3 students

How many students scored $\mathrm{A}(\geq 90)$ ?
5 students

The majority ( highest par ) of students scored between which \& which score ? 70 \& 80

What is the lowest score in the class ?
50
the lowest limit in the first class = lowest score

| gender | Grade |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | M | 10 | 5 | 20 | 35 |
|  | F | 15 | 10 | 30 | 55 |
|  | Total | 25 | 15 | 50 | $\mathbf{9 0}$ |

1-what Thais table a told?
Contingency: show relationship between two variables or more

## 2-How many variables?

## Two (grade, gender)

3- We chose one of probability find probability male or grade C?
$P(m$ or $C)=p(m)+p(C)-p(m, c)$
$=\frac{35}{90}+\frac{50}{90}-\frac{20}{90}=\frac{65}{90}=\frac{13}{18}$
4-find probability grad A or female?
$P(A$ or $F)=p(A)+p(F)-p(A, F)$
$=\frac{25}{90}+\frac{55}{90}-\frac{15}{90}=\frac{13}{18}$
5-find probability grade A or grad C?
$P(A$ or $C)=P(A)+P(C)-P(A, C)$
$=\frac{25}{90}+\frac{50}{90}-\frac{0}{90}=\frac{75}{90}$
M.e (mutely exclusive) if $p(A, B)=0$

6- are grade A and mate me?
$P(A$ and $m)=\frac{10}{90} \neq 0$ not $m$.e
7-are grade $B$ and $C$ me?
$P(B, C)=0$ yes m.e

## 8-find probability not getting A?

negative word in >> complement rule
$P(Z)=P(Z)=1-P(Z)$
$P(\bar{A})=1-P(A)=1-\frac{25}{90}=\frac{65}{90}$

## 9- Find probability not $\mathbf{C}$ ?

$P(C)=1-P(C)=1-\frac{50}{90}=\frac{40}{90}$
10-find probability female and grad B?
And: multiplication rule
$P(F$ and $B)=\frac{10}{90}$
11- Find probability male not grade $\mathbf{A}$ ?
$P(M$ or $A)=1-P(M$ or $A)$
$=1-(P(M)+p(A)-P(M, A)$
$=1-(35+25-10 / 90)=1-(50 / 90)=40 / 90$

## 12- Probability female given hear grade $A$ ?

Conditional: if, when, that $P(A)(B)=P(A, B) / P(B)$
$P(F / A)=P(F, A) / P(A)=15 / 25$
13 -find of male when Grade is $B$ ?
$P(M / B)=P(M, B) / P(B)=5 / 15$
14-if C what probability female?
start probability for required
$P(E / F)=P(F, E) / P(E)=30 / 50$
15- Male independent from $\mathbf{C}$ ?
$P(M, C)=P(M) * P(C)=35 / 90 * 50 / 90=0.22$
16- Are female independent from Grade B?
P (F, B) =10/90=0.11 لا يساوي

## $P(F) * P(B)=55 / 90 * 15 / 90=0.10$

## Not independent

25- a local bank reports that 80 percent of its customers maintain a checking account 60 percent have a savings account and 50 percent have both if a customer is chosen at random what is the probability the customer has either a checking or a saving account?

## $C>0.80 \quad S>0.60$ both $>C, S>0.50$

$P(C$ or $S)=P(C)+P(S)-P(C, S)$

## $=0.80+0.60-0.50=0.90$

$P(C$ or $S)=1-P(C$ or $S)=1-0,90=0.10$
4. three table listed below show (random variable) and their (probabilities) however only one of these is actually a probability distribution. A. which is it?

| $\mathbf{x}$ | $\mathbf{x}$ | $\mathbf{P}(\mathbf{X})$ |
| :---: | :---: | :---: |
| $\mathbf{5}$ | $\mathbf{3}$ |  |
| 10 | $\mathbf{3}$ |  |
| 15 | $\mathbf{2}$ |  |
| 20 | $\mathbf{4}$ |  |
| total | $\mathbf{1 2}$ |  |


| $X$ | $P(X)$ |
| :---: | :---: |
| 5 | .1 |
| 10 | .3 |
| 15 | .2 |
| 20 | .4 |
| total | 1 |


| $X$ | $P(X)$ |
| :---: | :---: |
| 5 | .5 |
| 10 | .3 |
| 15 | -2 |
| 20 | .4 |
| total | -0.8 |

probability distribution, fin the probability that x is:
(1)Exactly $15 \mathrm{P}(\mathrm{x}=15)=0.2$
(2) No more then 10

$=P(x=10)+p(x=5)=0.1+0.3=0.4$
3) More then 5

$$
P(X>5)=P(10)+P(15)+P(20)
$$

$$
=0.3+0.2+0.4=0.9
$$

(4) At least 12
px
أكبر من أو يساوي

12
$=P(15)+P(20)=0.2+0.4=0.6$

| $\mathbf{X}$ | $\mathbf{P}(\mathbf{X})$ | $\mathbf{X}^{\star} \mathbf{P}(\mathbf{X})$ | $\mathbf{X}^{\wedge} 2 \mathbf{P}(\mathbf{X})$ |
| :---: | :---: | :--- | :--- | :--- |
| 5 | .1 | $5(.1)=0.5$ | $5(.5)=2.5$ |
| 10 | .3 | $10(.3)=3$ | $10(3)=30$ |
| 15 | .2 | $15(.2)=3$ | $15(3)=45$ |
| 20 | .4 | $20(.4)=8$ | $20(8)=160$ |
| Total | 1 | 14.5 | 237.5 |

## 5) At most 7.5

## px(أصغرمن أو يساوي)

$=P(5)=0.1$
c- Compute the
mean, variance, and standard deviation of this distribution
Mean $=E(x)=14.5$
$X^{\wedge} 2^{*} P(X)$ - meanعVموعVariance $=V(X)=$
$237.5-14.5^{\wedge} 2=27.25$
V(X) جذر standard deviation $(X)=$
$=5.22$
$=\sqrt{27.25}=5.22$
20-in a binomial distribution $\mathrm{n}=12$ and $\mathrm{p}=.60$ find the following probabilities
A) $X=5$
$P(X=5)=12 C 5(0.60)^{\wedge} 5(1-0.60)^{\wedge} 12-5=0.10$
b) $x \leq 1$

## $P(x=1)+p(x=0)$

$12 \mathrm{C} 1(0.60)^{\wedge} 1(1-0.60)^{\wedge} 12-1+12 \mathrm{C} 0(0.60)^{\wedge} 0(1-0.60)^{\wedge} 12-0=0.0003$
c) $x \geq 12$

## P(12) اكبر من او يساويان

$=1-p(x<12)$
D) Find mean

Mean $=n \mathrm{p}=12(0.60)=7.2$
e) Find variance

Variance $=n p(1-p)=12(0.60)(1-0.60)=2.88$
32-in a Poisson distribution $\mathrm{p}=4$
A) What is the probability that $x=2$
$E^{\wedge}-4\left(4^{\wedge} 2 / 2!\right)=0.147$
b) what is the probability that $x$

## أصغر من أو يساوي1

$P(x=1)+p(x=0)$
$=e^{\wedge}-4\left(4^{\wedge} 1 / 1!\right)+\mathrm{e}^{\wedge}-4\left(4^{\wedge} 0 / 0!\right)=0.09$
c) What is the probability that $x>1$

1 1-p ( أصغر من أو يساوي 1
$=1-0.09=0.911$
d) Find mean
$=4$
e) Find variance
$=4$

Ms. Bergen is a loan officer at cost bank and trust .from her years of experiences. She estimates that the probability is .025 that an applicant will not be able to repay his or her instalment loan last month she made 40 loans
a) What is the probability that 3 loans will be defaulted?
$P(x)=40 C 3(0.025)^{3}(1-0.025)^{40-3}=0.06$
b) What is the probability that at least 3 loans will be defaulted?

3(اككر من او يساوي X)
$1-p(x<3)$
$P(X \geq 3)=1-P(x<3)$
the probability that a gulf air flight will arrive late is $20 \%$ a sample of 8 Gulf Air Flights was selected at random use the binomial distribution to find the
a) Expected number of Flights that arrive late and the standard deviation?
$\mathrm{P}($ arrive late $)=0.20$
$P($ not arrive late $)=0.80$
$\mu=n * p=8 * 20 \%=1.6$
b) Expected number of flights that will not arrive late
$\mu=n^{*} \mathrm{p}=8^{*} 0.8=6.4$
c) Probability that i) at least 2 flights will arrive late
$\mathrm{P}(\mathrm{x}=2$ or more $)$
$=1-((P=0)+(P=1))$
$=1-\left(8 \mathrm{C} 0(0.2)^{\wedge} 0(1-0.2)^{\wedge} 8-0\right)+8 \mathrm{C} 1(0.2)^{\wedge} 1(1-0.2)^{\wedge} 8-1$
$=1-(0.168+0.336)=1-0.504=0.496$
5 Flights will not arrive late
$\mathrm{nCx} P^{x}(1-\mathrm{P})^{\wedge} \mathrm{n}-\mathrm{x}$
$8 \mathrm{C} 5(0.8)^{\wedge} 5(1-0.8)^{\wedge} 8-5=1.47$
6.3 The sales of Lexus cars follows a Poisson distribution with a mean of 3 per day find the
a) Standard deviation of the number of Lexus cars sold per day

$$
\sigma=\sqrt{\mu}=\sqrt{3}=1.732
$$

b) Probability that i) at most 2 cars will be sold per day
$P(2$ or less $)=P(x=0)+P(x=1)+P(x=2)$

$$
\frac{3^{0} e^{-3}}{0 i}+\frac{3^{1} e^{-3}}{1 i}+\frac{3^{2} e^{-3}}{2 i}=0.0498+0.1494+0.2240=0.4232
$$

5 cars will be sold in 2 days
$3^{*} 2=6$
$P(x=5)$
$=0.161 \frac{6^{5} e^{-6}}{5 i}$

